An Optimal Design Problem of Infrastructure Consistent with Maintenance/Replacement Scheduling

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Abstract: Degrading of infrastructure not only results in lower service level but also in higher risk of earthquake damage. The maintenance/replacement should be well scheduled so as to maximize an economic value of infrastructure. On the other hand, the design of infrastructure should also follow the maximization principle. The design and the schedule of maintenance/replacement interact with each other. Then, we should optimize both of them in a unified framework. This paper proposes an optimal design problem consistent with maintenance/replacement scheduling. The problem is a type of bi-level programming, which contains an optimal scheduling problem at lower level and an optimal design at upper level. In the optimal scheduling, the degradation of infrastructure by fatigue is modeled as the Markov Chain Process. In the optimal design, the best material type is chosen. After we formulate a general framework for the problem, we apply it to a case in practice. The results suggest that the schedule and the design interact with each other and are very critical for the economic value of infrastructure.

1. INTRODUCTION

Degrading of infrastructure not only results in lower service level but also in higher risk of earthquake damage. The maintenance/replacement should be well scheduled so as to maximize an economic value of infrastructure. On the other hand, the design of infrastructure should also follow the maximization principle. The design and the schedule of maintenance/replacement interact with each other. Then, we should optimize both of them in a unified framework. This paper proposes an optimal design problem consistent with a maintenance/replacement strategy. The problem is a type of bi-level programming, which contains an optimal scheduling problem at lower level and an optimal design at upper level.

2. MODEL

Design of a structure is formalized as a bi-level programming. The upper level problem is to choose design variables such a material type, a size of member and a shape so as to maximize the economic value of an infrastructure. The programming at the upper level is,

$$V(X(0)) = \max_{d \in D_x} V(X(0), d) \tag{1}$$

where $V(\cdot)$ is the net present value of the infrastructure defined at the lower level, $X(0)$ the initial state of the infrastructure, d the design variable and D_x the feasible set of d.

The lower level problem is to find the rule of maintenance/replacement action $u(t)$ in any periods in time horizon indicated by $t \in \{0, \ldots, T\}$. Benefit/cost flow is dependent on the state of the infrastructure $X(t)$, action $u(t)$ and the design d. The net present value, which is the objective function at the upper level, is defined as

$$\begin{aligned} V(X(0), d) = \max_{u(t) \in U} E_{X(t)} \{ &\sum_{t=0}^{T} b(X(t), u(t), d)(1+\rho)^{-t} - \sum_{t=0}^{T} C(X(t), d)(1+\rho)^{-t} \\ &- \sum_{t=0}^{T} R(X(t), u(t), d)(1+\rho)^{-t} - \sum_{t=0}^{T} C_F(d) P_F(X(t))(1+\rho)^{-t} \} - I(d) \end{aligned} \tag{2.a}$$

s.t.

$$X(t) = F(X(t-1), u(t), d, \omega) \quad \text{for all} \quad t \in \{1, \ldots, T\}, \tag{2.b}$$

$$X(0) = \overline{X}(0) : \text{given} . \tag{2.c}$$

$b(\cdot)$ is the benefit in the period t. ρ is the social discount rate. $C(\cdot)$ is the operation cost. $R(\cdot)$ is the cost of maintenance/replacement action. $C_F(\cdot)$ is the recovery cost when damaged by earthquake and $P_F(\cdot)$ is the probability that the structure is damaged in the unit period. $I(\cdot)$ is the initial cost or the construction cost of the structure. The state of the infrastructure $X(t)$ evolves with the state equation in (2.b). The initial state $\overline{X}(0)$ is given.

The probability of damage by earthquake is assumed to be dependent on the state $X(t)$. If the structure is more deteriorated, then the probability is higher. The state equation in the above represents the stochastic process which includes the random factor ω.

The programming described by (2.a) – (2.c) is a kind of stochastic control problem. To solve it, the Bellman's principle of optimality is often applied and therefore the Bellman's equation is employed.

3. APPLICATION

3.1 Specification of the model

In this paper, we apply the model formalized in the previous section to design of a bridge. The bridge is a steel bridge whose initial cost can be simply estimated by total weight. The design is the choice of a material type $d \in \{1, 2, 3\}$, where 1 is the strong steel, 2 the normal steel and 3 the week steel. We first specify the state $X(t)$ in discrete form $X(t) = i(t) \in \{1, 2, 3, 4\}$. Then the action $u(t)$ is also specified as

$$u(i(t),t)=\begin{cases}0 \text{ for do nothing}\\ 1 \text{ for do the action}\end{cases} \quad (3)$$

Functions in (2.a) and (2.b) are specified as,

$$b(X(t),u(t),d)=b\psi(1+\lambda)^t \quad (4.a)$$

$$C(X(t),d)=\bar{c} \quad (4.b)$$

$$R(X(t),u(t),d)=\{b\psi(1+\lambda)^t l(i(t))+\alpha R(i(t))+\beta(w\upsilon(d)+A_1)\}u(i(t),t) \quad (4.c)$$

$$I(d)=w\upsilon(d)+A_2 \quad (4.d)$$

$$C_F(d)=w\upsilon(d)+A_3 \quad (4.e)$$

$$\text{and } P_F(X(t))=P_F(i(t)) \quad . \quad (4.f)$$

In (4.a), b is the annual benefit of a unit of traffic volume, ψ the initial annual traffic volume and λ the annual growth rate of traffic. In (4.b), the operation cost is simply assumed to be constant. In (4.c), the cost of maintenance/replacement action consists of $b\psi(1+\lambda)^t l(i(t))$ the reduction of benefit where $l(i(t))$ is the rate of stopping service due to the action, $R(i(t))$ the component dependent on the state $i(t)$ and $w\upsilon(d)+A_1$ dependent on the weight of the bridge. This cost is valid when the action is done $u(i(t),t)=1$.

The stochastic process denoted by the state equation in (2.b) is rewritten into the Markov Chain, whose transition probability matrix is given by

$$\mathbf{M}^{w.o.}(d)^{\frac{\psi(1+\lambda)^t}{\phi}}=(P_{i,j}^{w.o.}(d))_{i,j\in\{1,2,3,4\}} \text{ for nothing done} \quad (5.a)$$

$$\text{and } \mathbf{M}^w=(P_{i1}^w=1,P_{ij}^w=0)_{i\in\{1,2,3,4\},j\in\{2,3,4\}} \text{ for the action.} \quad (5.b)$$

(5.a) indicates that the probability matrix for the unit of traffic volume (a million cars per year) $\mathbf{M}^{w.o.}(d)$ must be multiplied with itself for the times $\frac{\psi(1+\lambda)^t}{\phi}$. Then in the year when traffic volume is large, the probability of deterioration becomes higher. The matrix in (5.b) means that whenever the action is done, the state always comes back to $i(t)=1$.

The Bellman equation for the model with the specifications is

$$
\begin{aligned}
&V(i(t)=i,d) \\
&= \max_{u(i(t),t)\in\{0,1\}} b\psi(1+\lambda)^t - \bar{c} - (b\psi(1+\lambda)^t l(i(t)) + \alpha R(i(t)) + \beta(w\upsilon(d)+A_1))u(i(t),t) \\
&\quad -(w\upsilon(d)+A_3)P_F(i(t)) \\
&\quad +\left(\frac{1}{1+\rho}\right)[\sum_{j\in\{1,2,3,4\}}\{(1-u(i(t),t))P_{i,j}^{w.o.}(d)+uu(i(t),t)P_{i,j}^{w}\}V(i(t+1)=j,d)]
\end{aligned} \quad . \tag{6}
$$

Solving the Bellman equation by backward induction (See Ueda and Kimoto(2003) and Judd(1998)), we can calculate the net present value in (2.a) for each design d .

The parameters and necessary information for specified function and matrices in (4.a)-(5.b) are listed in Table 1.

(a) Material type

	Strong (1)	Normal (2)	Week (3)
Spec	SM570	SM490	SM400
Cost $\upsilon(d)$	140,000(yen/ton)	110,000(yen/ton)	90,000(yen/ton)

(b) Parameters for benefit and cost

b	$\bar{c}$	w	$A_1 = A_2 = A_3$	$1/(1+\rho)$
60 (yen/car)	50000 (yen)	300 (ton)	250,000,000(yen)	0.96

(c) Parameters for maintenance/replacement action

State $i(t)$	Crack	Action	Cost of Action $\alpha R(i(t)) + \beta(w\upsilon(d)+A_1$
1	< 10 mm	TIG Processing	5,000,000 (yen)
2	10 mm - 15mm	Welding	10,000,000 (yen)
3	15mm – 30 mm	Steel Plate	25,000,000(yen)
4	30 mm <	Replacement	Initial Cost

(d) Parameters for earthquake risk

State $i(t)$	Probability of damage $P_F(i(t))$
1	0.05
2	0.1
3	0.2
4	0.35

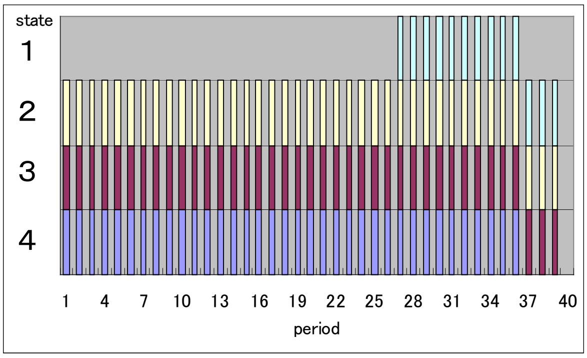
(e) Matrix for Markov Chain for the unit of traffic volume (a million cars/year)

$$
\mathbf{M}^{w.o.}(1)=\begin{bmatrix} 0.9930 & 0.0070 & & \\ & 0.9915 & 0.0085 & \\ & & 0.9800 & 0.0200 \\ & & & 1.0000 \end{bmatrix}
\quad
\mathbf{M}^{w.o.}(2)=\begin{bmatrix} 0.9846 & 0.0154 & & \\ & 0.9840 & 0.0160 & \\ & & 0.9576 & 0.0424 \\ & & & 1.0000 \end{bmatrix}
\quad
\mathbf{M}^{w.o.}(3)=\begin{bmatrix} 0.9630 & 0.0370 & & \\ & 0.9580 & 0.0420 & \\ & & 0.8670 & 0.1330 \\ & & & 1.0000 \end{bmatrix}
$$

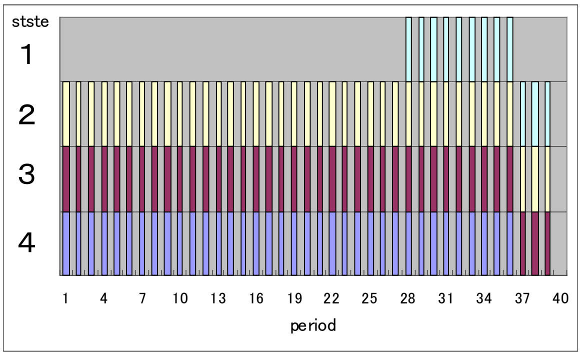
Table 1 Setting of parameters in case study

3.2 Results

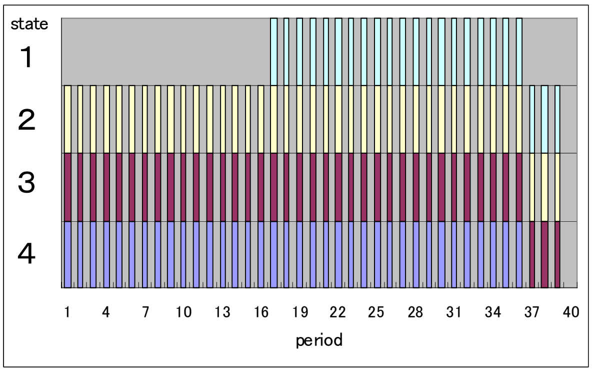
We first compare the optimal strategy of maintenance/replacement action for each material type when the annual growth rate of traffic is 6%. Figure 1 shows the strategy for each type. Each panel in the figure indicates that for example if the state in the period 4 becomes 2,3, or 4, then do the action, otherwise do noting. However, the strategy is not time invariant. Furthermore, we find differences in strategy between material types. Picking up the period 22, we find that if the state becomes 2,3, or 4, then do the action in the cases of design 1 (strong steel) and 2 (normal steel), and in contrast that if the state becomes 1, 2,3, or 4, then do the action in the cases of design 3 (week steel).



(a) Material type 1 (strong steel)



(b) Material type 2 (normal steel)



(c) Material type 3 (week steel)

Figure 1 Comparison of maintenance/replacement strategy between material types

The differences in maintenance/replacement strategy also result in differences in the net present value of the bridge as shown in Figure 2. The material type 1 (strong steel) indicates the highest economic value, 2 (normal steel) the second and 3 (week steel) the lowest. As a solution of the upper level programming, the optimal design as material choice is the type 1 (strong steel) in this case.

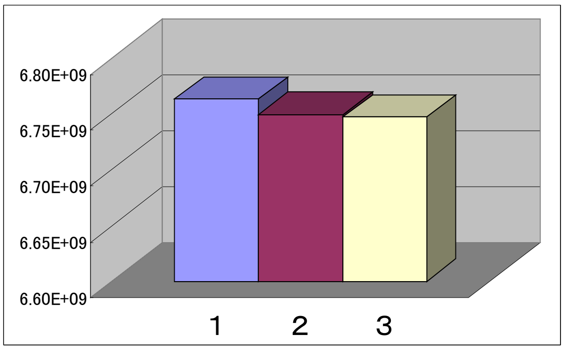


Figure 2 Comparison of the economic value of the bridge between material types for 6% traffic growth

As the society becomes more aged or has less population, the traffic volume may decrease. It is of interest that we examine the case that the annual growth rate of traffic is negative, –3%. Figure 3 shows the net present value of the bridge for each material type. In contrast to the case of 6% as already examined, the material type 3 indicates the highest economic value. The optimal design has varied from the previous case.

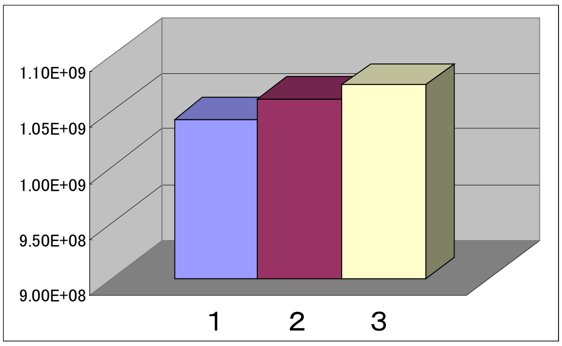


Figure 3 Comparison of the economic value of the bridge between material types for -3% traffic growth

Throughout cases in various setting of parameters, we have found that the optimal design and strategy of maintenance/replacement action interact with each other. Although the implications are still case-specific, the approach of the bi-level programming has been proved to be useful in the economic design of the infrastructure.

4. CONCLUDING REMARKS

We have proposed a bi-level programming model for the economic design of an infrastructure. The results of case studies suggest that the schedule and the design interact with each other and are very critical for the economic value of infrastructure.

We should try to enhancement of the model and examination of the model throughout more and more case studies. In particular, stochastic process of deterioration must be modeled more carefully.

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